

Modelling Tachyon Cosmology with Non-Minimal Derivative Coupling to Gravity

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Abstract – We study a tachyon model with non-minimal derivative coupling to gravity in the Friedmann-Robertson-Walker (FRW) flat cosmology. We propose the special re-definition of the tachyon field which allows us to represent tachyon field equation formally coinciding with its usual form but with re-defined Hubble parameter. Two first integrals for the model equations are obtained that can essentially simplify both further analysis and analytical solving for the model. These integrals become the trivial identities in the case of minimal coupling. The effective energy density and pressure of the tachyon field are obtained, and the necessary condition of the possibility for this model to expand with acceleration is derived.

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1 Introduction

According to recent observational data, such as SNe Ia [1]-[3], WMAP [4], SDSS [5] etc., our Universe is flat and undergoing a phase of the accelerated expansion which started about five billion years ago. They also suggest that it consists of about 70% dark energy (DE) with negative pressure, 30% dust matter (cold dark matter plus baryons), and negligible radiation. Dark energy has been one of the most active fields in modern cosmology. In modern cosmology of DE, the equation of state parameter $p = \gamma\rho$ plays an important role, where p and ρ are its pressure and energy density, respectively. To accelerate the expansion, the equation of state must satisfy $\gamma < -1/3$. As a possible solution to the DE problem various dynamical models of DE have been proposed, such as quintessence [6], tachyon [7], phantom [8] and quintom [9], and so forth. Besides, other proposals on DE include interacting dark energy models [10, 11], braneworld models [12], and holographic dark energy models [13, 14], etc. What distinguishes the tachyon action from the standard Klein-Gordon form for scalar field is that the tachyon action is non-standard and is of the Dirac-Born-Infeld form [7].

In recent years, an alternative possibility of having an effective scalar field theory governed by a Lagrangian containing a non canonical kinetic term $L = -V(\phi)F(X)$, where $X = (1/2)\partial_\mu\phi\partial^\mu\phi$ has been proposed. One of the most studied forms for $F(X)$ is $F(X) = \sqrt{1 - 2X}$. Such a model can lead to late time acceleration of the Universe and is called k-essence. This field can also give rise to inflation in early universe and is called k-inflation. This type of field can naturally arise in string theory and can be very interesting in cosmological context [9].

The alternative approach to the problem of accelerated expansion is the consideration of various modifications of the gravitation theory. Among such modifications, the theories with non-minimal coupling of a field to gravity are especially attractive. For instance, scalar tensor theories are generalization of the minimally coupled scalar field theories in a sense that here the scalar field is non-minimally coupled with the gravity sector of the action i.e with the Ricci scalar R . In these theories, the scalar field participates in the gravitational interaction, unlike its counterpart in the minimally coupled case where it behaves as a non gravitational source like any other matter field.

Non-minimally coupled to gravity tachyon fields in cosmology have been considered earlier, for example, in papers [15]-[20]. In our paper we present the cosmological paradigm with non-minimal *derivative* coupling between a tachyon field and the curvature. Interest to such a type of coupling between different fields and gravity is confirmed by increasing number of publications devoted to

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this direction of research (see, for example, [21]-[24]). In addition to the derivation of the main equations of the tachyon model with non-minimal derivative coupling, we find the first integrals of the model and conditions for accelerated expansion of the model.

2 Main model equations

Let us consider a cosmological theory of a tachyon field with non-minimal derivative coupling to curvature. In general, one could have various forms of such coupling. For instance in the case of two derivatives, one could have the terms $\xi R\phi_{,\mu}\phi^{,\mu}$ and $\zeta R_{\mu\nu}\phi^{,\mu}\phi^{,\nu}$, where the coefficients ξ, ζ are the coupling parameters [21]. Therefore, we start with the following action:

$$S = \int \left(-\frac{R}{2\kappa} - V(\phi) \sqrt{1 - \mathcal{G}^{\mu\nu} \partial_\mu \phi \partial_\nu \phi} \right) \sqrt{-g} d^4x, \quad (1)$$

where $\kappa = 8\pi G = M_P^{-2}$ is the gravitational constant, M_P is the Planck mass, $V(\phi)$ is a potential of the tachyon field ϕ ,

$$\mathcal{G}^{\mu\nu} = \alpha g^{\mu\nu} + \xi R g^{\mu\nu} + \zeta R^{\mu\nu}. \quad (2)$$

Here α, ξ and ζ are the dimensional constants of the tachyon field coupling with gravity. For simplicity, we consider a spatially flat FRW cosmological model with the space-time interval

$$ds^2 = N^2(t) dt^2 - a^2(t) (dr^2 + r^2 d\Omega^2), \quad (3)$$

where $a(t)$ is a scale factor, and $N(t)$ is a lapse function. Assuming homogeneity of the tachyon field, i.e. $\phi = \phi(t)$, and calculating for the interval (3)

$$R_{00} = -\frac{3\ddot{a}N - \dot{a}\dot{N}}{aN}, \quad R = -6\frac{a\ddot{a}N - a\dot{a}\dot{N} + \dot{a}^2N}{a^2N^3}, \quad (4)$$

from (1) one can obtain the following expression for the kinetic term $X = \mathcal{G}^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$:

$$X = \dot{\phi}^2 \left[\frac{\alpha}{N^2} - 3(2\xi + \zeta) \frac{\ddot{a}}{aN^4} + (6\xi + \zeta) \frac{\dot{a}\dot{N}}{aN^5} - 6\xi \frac{\dot{a}^2}{a^2N^4} \right]. \quad (5)$$

On account of the latter and metrics (3) we obtain from action (1) the effective Lagrangian of the model in the form:

$$L = \frac{3}{8\pi G} \left(\frac{a^2\ddot{a}}{N} + \frac{a\dot{a}^2}{N} - \frac{a^2\dot{a}\dot{N}}{N^2} \right) - a^3NV(\phi)\sqrt{1-X}. \quad (6)$$

As one can see, the term proportional to the second derivative \ddot{a} is in X of (5), i.e. in the term non-linearly included in Lagrangian (6). It means that the dynamical equations of the model will contain the scale factor derivatives of the order higher than two. As it was noted in [21] for the canonical non-minimal scalar field theory, the appearance of such derivatives can be avoided due to the particular choice of coupling constants in (2) without loss of generality. Therefore, we also put $\xi = -\frac{1}{2}\zeta$. The latter means that tensor (2) is expressed through the metrics tensor and the Einstein tensor as

$$\mathcal{G}^{\mu\nu} = \alpha g^{\mu\nu} + \zeta \left(R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R \right).$$

The corresponding expression for X in (5) can be re-written as follows:

$$X = \frac{\dot{\phi}^2}{N^2} Y, \quad Y = \alpha - 2\zeta \frac{\dot{a}\dot{N}}{aN^3} + 3\zeta \frac{\dot{a}^2}{a^2N^2}. \quad (7)$$

Integrating the first term in Lagrangian (6) by parts and substituting (7), we obtain the following effective Lagrangian of the model:

$$L = -\frac{3}{8\pi G} \frac{a\dot{a}^2}{N} - a^3V(\phi)\sqrt{N^2 - \dot{\phi}^2 Y}. \quad (8)$$

Then taking into account the explicit form of Y from (7) and putting the gauge $N = 1$, we obtain by variation over ϕ, N and a the following main equations of the model:

$$\frac{\ddot{\phi}Z}{1 - \dot{\phi}^2 Z} + 3H\dot{\phi}\left(Z + \zeta\dot{H}\frac{2 - \dot{\phi}^2 Z}{1 - \dot{\phi}^2 Z}\right) + \frac{V'}{V} = 0, \quad (9)$$

$$\frac{3}{8\pi G}H^2 = V\frac{1 - \zeta(\dot{\phi}^2\dot{H} + 2\dot{\phi}\ddot{\phi}H)}{\sqrt{1 - \dot{\phi}^2 Z}} - V'\zeta\frac{\dot{\phi}^3 H}{\sqrt{1 - \dot{\phi}^2 Z}} - V\zeta\dot{\phi}^2 H\frac{\dot{\phi}\ddot{\phi}Z + 3\zeta\dot{\phi}^2 H\dot{H}}{(1 - \dot{\phi}^2 Z)^{3/2}}, \quad (10)$$

$$\begin{aligned} \frac{1}{8\pi G}(2\dot{H} + 3H^2) &= V\sqrt{1 - \dot{\phi}^2 Z} + V\zeta\frac{3\dot{\phi}^2 H^2 + (\dot{\phi}^2\dot{H} + 2\dot{\phi}\ddot{\phi}H)}{\sqrt{1 - \dot{\phi}^2 Z}} + \\ &+ V'\zeta\frac{\dot{\phi}^3 H}{\sqrt{1 - \dot{\phi}^2 Z}} + V\zeta\dot{\phi}^2 H\frac{\dot{\phi}\ddot{\phi}Z + 3\zeta\dot{\phi}^2 H\dot{H}}{(1 - \dot{\phi}^2 Z)^{3/2}}, \end{aligned} \quad (11)$$

where $Z = Y|_{N=1}$, that is

$$Z = \alpha + 3\zeta H^2, \quad (12)$$

where $H = \dot{a}/a$ is the Hubble parameter, $V' = dV/d\phi$.

By addition of equations (10) and (11), one can obtain:

$$\frac{1}{4\pi G}(\dot{H} + 3H^2) = V\sqrt{1 - \dot{\phi}^2 Z} + V\frac{1 + 3\zeta H^2 \dot{\phi}^2}{\sqrt{1 - \dot{\phi}^2 Z}}. \quad (13)$$

Transforming the tachyon field $\phi(t) \rightarrow \Phi(t)$ according to

$$\dot{\phi}\sqrt{Z} = \dot{\Phi}, \quad (14)$$

and using the consequent expression

$$\dot{\phi}\ddot{\phi}Z + 3\zeta\dot{\phi}^2 H\dot{H} = \dot{\Phi}\ddot{\Phi}, \quad \frac{dV}{d\phi} = \sqrt{Z}\frac{dV}{d\Phi}, \quad (15)$$

we can re-write (9) for the tachyon field Φ as follows:

$$\frac{\ddot{\Phi}}{1 - \dot{\Phi}^2} + 3H\dot{\Phi}\left(1 + \frac{\zeta\dot{H}}{\alpha + 3\zeta H^2}\right) + \frac{V'}{V} = 0, \quad (16)$$

where $V' = \frac{dV}{d\Phi}$.

Equation (16) differs from the tachyon equation in the case of minimal coupling only in the presence of the second term in brackets, and reduces to it as $\zeta = 0$ or $\dot{H} = 0$. The latter means either the minimally coupled tachyon field or the constancy of the Hubble parameter. In view of Y from (7) for $H = \text{constant}$, only re-scaling of parameter α in Lagrangian (8) is required to remain in the minimally coupled theory. Complete similarity of equation (16) with the similar equation in the case of minimal coupling and $H \neq \text{constant}$ can be obtained by substitution

$$f = aZ^{1/6}, \quad \mathcal{H} = \frac{\dot{f}}{f} = H + \frac{\dot{Z}}{6Z}, \quad (17)$$

which transforms equation (16) as follows:

$$\frac{\ddot{\Phi}}{1 - \dot{\Phi}^2} + 3\mathcal{H}\dot{\Phi} + \frac{V'}{V} = 0. \quad (18)$$

Substituting (14), (15) into equations (10) and (13) we can obtain the model equations as follows:

$$\frac{3}{8\pi G}H^2 = \frac{V}{\sqrt{1 - \dot{\Phi}^2}} + V\zeta\frac{3\mathcal{H}\dot{\Phi}^2(\dot{\Phi}^2 H Z^{-1}) - \frac{d}{dt}(\dot{\Phi}^2 H Z^{-1})}{\sqrt{1 - \dot{\Phi}^2}}, \quad (19)$$

$$\frac{1}{4\pi G}(\dot{H} + 3H^2) = V\sqrt{1 - \dot{\Phi}^2} + V \frac{1 + 3\zeta H^2 Z^{-1} \dot{\Phi}^2}{\sqrt{1 - \dot{\Phi}^2}}, \quad (20)$$

The set of equations (18) - (20) is rather complicated to be analyzed and exactly solved. However, the situation is essentially simplified if we take into account the existence of the first integrals. In the next section, we get two such integrals. We also briefly discuss the effective equation of state for the model.

3 The first integrals and equation of state

First of all, we will show that equation (11) is a differential consequence of the following first integral:

$$\frac{3}{8\pi G}H^2 - V \frac{1 + 3\zeta H^2 \dot{\Phi}^2}{\sqrt{1 - \dot{\Phi}^2} Z} = \frac{K}{a^3}, \quad (21)$$

where $K = \text{constant}$, and the tachyon field equation (9). Indeed, multiplying equation (21) by a^3 and differentiating the result with respect to time, we have:

$$\begin{aligned} \frac{3}{8\pi G}(\dot{a}^3 + 2a\dot{a}\ddot{a}) &= V'\dot{\Phi}a^3 \frac{1 + 3\zeta H^2 \dot{\Phi}^2}{\sqrt{1 - \dot{\Phi}^2} Z} + 3Va^2\dot{a} \frac{1 + 3\zeta H^2 \dot{\Phi}^2}{\sqrt{1 - \dot{\Phi}^2} Z} + \\ &+ 3Va^3\zeta \frac{2H\dot{H}\dot{\Phi}^2 + 2H^2\ddot{\Phi}\dot{\Phi}}{\sqrt{1 - \dot{\Phi}^2} Z} + Va^3(1 + 3\zeta H^2 \dot{\Phi}^2) \frac{\ddot{\Phi}\dot{\Phi}Z + \dot{\Phi}^2 3\zeta H\dot{H}}{(1 - \dot{\Phi}^2 Z)^{3/2}} = 0. \end{aligned}$$

Dividing the latter by a^3 and using the identity $\ddot{a}/a = \dot{H} + H^2$, we get:

$$\begin{aligned} \frac{3H}{8\pi G}(2\dot{H} + 3H^2) &= 3H \left\{ V\sqrt{1 - \dot{\Phi}^2} Z + V\zeta \frac{3\dot{\Phi}^2 H^2 + (\dot{\Phi}^2 \dot{H} + 2\ddot{\Phi}\dot{\Phi}H)}{\sqrt{1 - \dot{\Phi}^2} Z} + V'\zeta \frac{\dot{\Phi}^3 H}{\sqrt{1 - \dot{\Phi}^2} Z} + \right. \\ &+ V\zeta \dot{\Phi}^2 H \frac{\ddot{\Phi}\dot{\Phi}Z + 3\zeta \dot{\Phi}^2 H\dot{H}}{(1 - \dot{\Phi}^2 Z)^{3/2}} \left. \right\} + \left[V \frac{\ddot{\Phi}\dot{\Phi}Z + 3\zeta \dot{\Phi}^2 H\dot{H}}{(1 - \dot{\Phi}^2 Z)^{3/2}} + V \frac{3\zeta H\dot{H}\dot{\Phi}^2}{\sqrt{1 - \dot{\Phi}^2} Z} + \frac{3VH}{\sqrt{1 - \dot{\Phi}^2} Z} + \right. \\ &\left. + \frac{V'\dot{\Phi}}{\sqrt{1 - \dot{\Phi}^2} Z} - 3VH\sqrt{1 - \dot{\Phi}^2} Z \right]. \end{aligned}$$

The expression in the square brackets is zero in view of equation (9). On cancelling out $3H$ in the remaining equation, one is left with equation (11). In terms of re-defined field (14), the first integral (21) can be written as:

$$\frac{3}{8\pi G}H^2 = V \frac{1 + 3\zeta H^2 Z^{-1} \dot{\Phi}^2}{\sqrt{1 - \dot{\Phi}^2}} + \frac{K}{a^3}, \quad (22)$$

In the case of minimal coupling with $\alpha = 1$, $\zeta = 0$, $Z = 1$, equation (21) coincides with equation (10) considered under the same conditions and zero constant of integration $K = 0$. The latter follows from the existence of only two independent equations in minimally coupled model.

From equations (20) and (22), the following consequence can be obtained:

$$\frac{3}{4\pi G} \frac{\ddot{a}}{a} \equiv \frac{3}{4\pi G}(\dot{H} + H^2) = V \frac{2 - 3(1 + \zeta H^2 Z^{-1})\dot{\Phi}^2}{\sqrt{1 - \dot{\Phi}^2}} - 4 \frac{K}{a^3}. \quad (23)$$

This equation is convenient to analyze the possibility of accelerated expansion $\ddot{a} > 0$.

On the other hand, comparing the right sides of (20) and (22) with similar equations for a perfect fluid (i.e. $-(\rho + 3p)$ and ρ , respectively), we can obtain the following formal expressions for the effective energy density and pressure:

$$\rho = V \frac{1 + 3\zeta H^2 Z^{-1} \dot{\Phi}^2}{\sqrt{1 - \dot{\Phi}^2}} + \frac{K}{a^3}, \quad p = -V \sqrt{1 - \dot{\Phi}^2} + \frac{4}{3} \frac{K}{a^3}. \quad (24)$$

These expressions imply the effective equation of state of the form:

$$\gamma \equiv \frac{p}{\rho} = - \frac{3a^3 V(1 - \dot{\Phi}^2) - 4K\sqrt{1 - \dot{\Phi}^2}}{3a^3 V(1 + 3\zeta H^2 Z^{-1} \dot{\Phi}^2) + 3K\sqrt{1 - \dot{\Phi}^2}}, \quad (25)$$

which can be written as

$$\gamma_{K=0} = - \frac{1 - \dot{\Phi}^2}{1 + 3\zeta H^2 Z^{-1} \dot{\Phi}^2} \quad (26)$$

in the case of zero constant $K = 0$.

From the condition of accelerated expansion, $\gamma_{K=0} < -1/3$, we have the following inequality

$$3(1 + \zeta H^2 Z^{-1})\dot{\Phi}^2 < 2, \quad (27)$$

This condition can also be obtained from $\ddot{a} > 0$ in equation (23). It should be noted that due to uncertainty in the sign of constant $K \neq 0$ in (25) the additional terms $\sim K$ can significantly change equation of state in each direction from the critical value $\gamma = -1$.

Another first integral follows from the equality of the right-hand-sides of equations (19) and (22). After the substitution of $3\mathcal{H}\dot{\Phi}$ from equation (18), the result can be written as

$$\zeta \frac{d}{dt} \ln \left[\frac{\dot{\Phi}^2 H Z^{-1} a^3 V}{\sqrt{1 - \dot{\Phi}^2}} \right] + \frac{K\sqrt{1 - \dot{\Phi}^2}}{a^3 \dot{\Phi}^2 H Z^{-1} V} = 0.$$

Hence, we obtain the following first integral:

$$\zeta \left(\frac{\dot{\Phi}^2 H a^3}{Z} \right) \frac{V}{\sqrt{1 - \dot{\Phi}^2}} = -K t + \zeta L, \quad (28)$$

where L is an arbitrary constant. In the case of minimal coupling, i.e. for $\zeta = 0$, we obtain $K = 0$, that is a trivial identity. The latter means the absence of the first integral of the form (28) in the case of minimal coupling to gravity. From (28), it also follows that $K = L = 0$ for $V = 0$ and / or $\dot{\Phi} = 0$. In the first case, we obtain from (24) the expected result : $\rho = p = 0$. In the second case from (26), we obtain the quasi vacuum state $\gamma = -1$ with $p = -\rho = -V_0 = \text{constant}$, because of equation (18) implies the constancy of potential $V = V_0$ for $\dot{\Phi} = 0$.

Possessing these first integrals, we can try to construct some exact solutions of the model. One can offer a possible way for that which consists of the following. For simplicity, we assume that $K = 0$ and substitute expression $V/\sqrt{1 - \dot{\Phi}^2}$ from (28) into equation (22). As a result, we have:

$$\frac{3}{8\pi G} H^2 = \frac{LZ}{a^3 \dot{\Phi}^2 H} \left(1 + 3\zeta H \frac{\dot{\Phi}^2 H}{Z} \right). \quad (29)$$

By specifying a certain dependence $\dot{\Phi}(t)$, one can find $H(t)$ and $a(t)$ from this equation. Let us note that due to (24) equation (29) can be expressed in terms of the original field ϕ in (14) as

$$\frac{3}{8\pi G} \dot{a}^3 = L \left(\frac{1}{\dot{\phi}^2} + 3\zeta \frac{\dot{a}}{a} \right).$$

After that, it becomes possible to obtain a potential V from (28) or (18). Obviously, we have to be confident that these two results for V will coincide. So differentiating (28) and partially replacing the terms with the help of equation (18), after simple manipulations one can show that these two solutions for $V(\Phi)$ will coincide if the following equality is valid:

$$\frac{d}{dt} \ln \left(\frac{\dot{\Phi}^2 H a^3}{Z} \right) = 3\mathcal{H}\dot{\Phi}^2. \quad (30)$$

Substituting $\frac{\dot{\Phi}^2 H a^3}{Z}$ from (28) with $K = 0$ into (30) we have

$$\frac{d}{dt} \left(\ln \frac{V}{\sqrt{1 - \dot{\Phi}^2}} \right) + 3\mathcal{H}\dot{\Phi}^2 = 0. \quad (31)$$

Obviously, the latter is just the tachyon field equation (18).

Of course, there exist different approaches for finding the exact solutions but (30) always has to be taken into account in the case of non-minimal coupling. We have to emphasize that there is no need to verify this equation in the case of minimal coupling because the first integral (28) does not exist at all as $\zeta = 0$. Nevertheless, the minimally coupled analog of equation (31) is valid, that can be seen from the usual tachyon equation:

$$\frac{\ddot{\phi}}{1 - \dot{\phi}^2} + 3H\dot{\phi} + \frac{V'}{V} = 0 \Leftrightarrow \frac{d}{dt} \left(\ln \frac{V}{\sqrt{1 - \dot{\phi}^2}} \right) + 3H\dot{\phi}^2 = 0. \quad (32)$$

It is appropriate mention here that, on view of the Friedmann equation

$$\frac{3}{8\pi G} H^2 = \frac{V}{\sqrt{1 - \dot{\phi}^2}}, \quad (33)$$

and (32), we can find the following formulas:

$$\dot{\phi}^2(t) = \frac{2}{3} \frac{d}{dt} \left(\frac{1}{H} \right), \quad V(t) = \frac{3}{8\pi G} H^2 \sqrt{1 - \frac{2}{3} \frac{d}{dt} \left(\frac{1}{H} \right)},$$

which allows us, starting with some $H(t)$ given, to get a wide class of exact solutions for the case of minimal coupling. The alternative approach serving this same purpose is recently proposed in [25]. Unfortunately, it is not so easy to obtain the exact solutions in the case of non-minimal derivative coupling.

4 Conclusion

Thus, we have studied the spatially flat FRW tachyon model with non-minimal derivative coupling to gravity. We have obtained the main equations of the model in the form (9) - (11). Besides, it was suggested that the specific representation of tachyon field (14) could allow us to get the equation formally identical to a similar equation with minimal coupling (18), but with the re-defined Hubble parameter (17). Two first integrals (22), (28) for the model equations have been found which could essentially simplify both analysis and solution, and which became the identities in the case of minimal coupling. The set of main equations governing the model consists of (22), (28) and (30) at least in the case of $K = 0$. Thus, we have proposed a method for constructing exact solutions for the tachyon model in FRW cosmology based on the first integrals obtained above. In the framework of our model, the effective energy density and pressure of the tachyon field (24) have been found. The conditions for the cosmic accelerated expansion (27) have been found as well. Further details and consequences of the model considered here are in progress.

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